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Brief paper

Output feedback stabilization of the Korteweg–de Vries equation☆

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ABSTRACT

This paper presents an output feedback control law for the Korteweg–de Vries equation. The control design is based on the backstepping method and the introduction of an appropriate observer. The local exponential stability of the closed-loop system is proven. Some numerical simulations are shown to illustrate this theoretical result.

1. Introduction

The Korteweg–de Vries (KdV) equation was introduced in 1895 to describe approximatively the behavior of long waves in a water channel of relatively shallow depth. Since then, this equation has attracted a lot of attention due to fascinating mathematical features and a number of possible applications.

From a control viewpoint, the KdV system also presents amazing behaviors. Surprisingly, by considering different boundary actuators on a bounded interval $[0, L]$, we get control results of different nature. Roughly speaking, the system is exactly controllable when the control acts from the right endpoint $x = L$ (Cerpa, Rivas, & Zhang, 2013; Rosier, 1997), and null-controllable when the control acts from the left endpoint $x = 0$ (Carreño & Guerrero, 2015; Glass & Guerrero, 2008; Guilleron, 2014), whether the control is Neumann or Dirichlet type.

Due to this kind of phenomena, the control properties of this nonlinear dispersive partial differential equation have been deeply studied. However, there still are many open questions. See Cerpa (2014), Marx, Cerpa, Prieur, and Andrieu (2017) and Rosier and Zhang (2009), and the references therein.

In this article we focus on the boundary stabilization problem for the KdV equation with a control acting on the left Dirichlet boundary condition. The studied system can be written as follows

$$\begin{cases} u_t + u_x + u_{xxx} + uu_x = 0, \\ u(t,0) = \kappa(t), \ u_x(t,L) = 0, \ u_{xx}(t,L) = 0, \\ u(0,x) = u_0(x), \end{cases} \tag{1}$$

where $\kappa = \kappa(t)$ denotes the boundary control input and $u_0 = u_0(x)$ is the initial condition. Concerning the stability when no control is applied ($\kappa = 0$), it is known that the linear system is asymptotically stable (see Tang and Krstic (2013, Lemma 3)). We aim here to design an output feedback control in order to get the exponential stability of the closed-loop system.

Some full state feedback controls have already been designed in the literature for KdV systems. When the control acts on the right endpoint, we find (Cerpa & Crépeau, 2009) where a Gramian-based method is applied, and (Coron & Lü, 2014) where some suitable integral transforms are used. In Cerpa and Coron (2013), Marx and Cerpa (2014) and Tang and Krstic (2013), the authors use the backstepping method to design feedback controllers acting on the left endpoint of the interval.

However, in most cases, we have no access to measure the full state of the system. Thus, it is more realistic to design an output feedback control, i.e., a feedback law depending only on some partial measurements of the state.

For autonomous linear finite-dimensional systems, the separation principle holds. Thus, stabilizability and observability assumptions are sufficient to ensure the stability of the closed-loop system. In other words, if there exists a controller, which asymptotically stabilizes the origin of the system and an observer which converges asymptotically to the state system, the output feedback built from this observer and this state feedback asymptotically stabilizes the origin of the system. The case of autonomous nonlinear finite-dimensional systems depends critically on the structure of the

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system. We can only hope having a semi-global result (see e.g Teel and Praly (1994)). In a PDE framework, this principle, even for linear systems, is no longer true and the stability of the closed-loop system is not guaranteed.

The basic question to state the problem is which kind of measurements we are going to consider, being the boundary case the most challenging one. In Marx and Cerpa (2014) we consider the linear KdV equation with boundary conditions

$$u(t,0)=\kappa(t),\ u(t,L)=0,\ u_x(t,L)=0. \tag{2}$$

In that paper, we see that this system is not observable from the output $y(t)=u_x(t,0)$ for some values of L. However, we design an output feedback law exponentially stabilizing the system for the output given by $y(t)=u_{xx}(t,L)$. Thus, we see that the choice of the output is crucial. In Hasan (2016), the same controller has been applied to the nonlinear Korteweg–de Vries equation.

In this paper, we will consider the nonlinear KdV equation (1) with measurement

$$y(t)=u(t,L). \tag{3}$$

Independently to Marx and Cerpa (2014) and to the present paper, Tang and Krstic have developed the same program for similar linear KdV equations. Full state (Tang & Krstic, 2013) and output state (Tang & Krstic, 2015) feedback controls are designed by using the backstepping method.

This paper is organized as follows. In Section 2, we state our main result. Section 3 is devoted to recall the feedback control designed in Cerpa and Coron (2013). The observer is built in Section 4. In Section 5, the well-posedness and the exponential stability of the linear closed-loop controller–observer system is proven. In Section 6, we prove both the local well-posedness and the exponential stability of the nonlinear closed loop controller–observer system. Some numerical simulations are presented in Section 7. Finally, Section 8 states some conclusions.

2. Main result

Based on Krstic and Smyshlyaev (2008) and Smyshlyaev and Krstic (2005), we built an observer for (1). More precisely, we define, for some appropriate function $p_1(x)$, the following copy of the plant with a term depending on the observation error

$$\begin{cases} \hat{u}_t+\hat{u}_x+\hat{u}_{xxx}+\hat{u}\hat{u}_x+p_1(x)[y(t)-\hat{u}(t,L)]=0,\\ \hat{u}(t,0)=\kappa(t),\ \hat{u}_x(t,L)=\hat{u}_{xx}(t,L)=0,\\ \hat{u}(0,x)=\hat{u}_0(x). \end{cases} \tag{4}$$

As mentioned in Section 1, our main result is the local stabilization of the KdV equation by using the output (3), as stated in the following theorem whose proof is given in Section 6.

Theorem 1. *For any $\lambda>0$, there exist an output feedback law $\kappa(t):=\kappa(\hat{u}(t,x))$, a function $p_1=p_1(x)$, and two constants $C>0$, $r>0$ such that for any initial conditions $u_0,\hat{u}_0\in L^2(0,L)$ satisfying*

$$\|u_0\|_{L^2(0,L)}\le r,\quad \|\hat{u}_0\|_{L^2(0,L)}\le r, \tag{5}$$

the solution of (1)–(3)–(4) satisfies

$$\begin{aligned}&\|u(t,\cdot)-\hat{u}(t,\cdot)\|_{L^2(0,L)}+\|\hat{u}(t,\cdot)\|_{L^2(0,L)}\\ &\quad\le Ce^{-\lambda t}\Big(\|u_0-\hat{u}_0\|_{L^2(0,L)}+\|\hat{u}_0\|_{L^2(0,L)}\Big),\quad \forall t\ge 0.\end{aligned} \tag{6}$$

Remark 1. Notice that from this theorem we get the exponential decreasing to 0 of the L^2-norm of the solution $u=u(t,x)$ provided that the L^2-norm of the initial conditions of the plant and the observer are sufficiently small.

3. Control design

The backstepping design applied here is based on the linear part of the equation. Thus, we consider the control system linearized around the origin

$$\begin{cases} u_t+u_x+u_{xxx}=0,\\ u(t,0)=\kappa(t),\ u_x(t,L)=u_{xx}(t,L)=0,\\ u(0,x)=u_0(x), \end{cases} \tag{7}$$

and the linear observer

$$\begin{cases} \hat{u}_t+\hat{u}_x+\hat{u}_{xxx}+p_1(x)[y(t)-\hat{u}(t,L)]=0,\\ \hat{u}(t,0)=\kappa(t),\ \hat{u}_x(t,L)=\hat{u}_{xx}(t,L)=0,\\ \hat{u}(0,x)=\hat{u}_0(x). \end{cases} \tag{8}$$

The standard method of output feedback design follows a three-step strategy. We first design the full state feedback control. Next, we built the observer. Finally, we prove that plugging the observer state into the feedback law stabilizes the closed loop system.

In Cerpa and Coron (2013) the following Volterra transformation is introduced

$$w(x)=\Pi(u(x)):=u(x)-\int_x^L k(x,y)u(y)dy. \tag{9}$$

The function k is chosen such that $u=u(t,x)$, solution of (7) with control

$$\kappa(t)=\int_0^L k(0,y)u(t,y)dy, \tag{10}$$

is mapped into the trajectory $w=w(t,x)$, solution of the linear system

$$\begin{cases} w_t+w_x+w_{xxx}+\lambda w=0,\\ w(t,0)=w_x(t,L)=w_{xx}(t,L)=0, \end{cases} \tag{11}$$

which is exponentially stable for $\lambda>0$, with a decay rate at least equal to λ.

The kernel function $k=k(x,y)$ is characterized by

$$\begin{cases} k_{xxx}+k_{yyy}+k_x+k_y=-\lambda k,\ \text{in } \mathcal{T},\\ k(x,L)+k_{yy}(x,L)=0,\ \text{in } [0,L],\\ k(x,x)=0,\ \text{in } [0,L],\\ k_x(x,x)=\dfrac{\lambda}{3}(L-x),\ \text{in } [0,L], \end{cases} \tag{12}$$

where $\mathcal{T}:=\{(x,y)/x\in[0,L],y\in[x,L]\}$. The solution k to (12) exists and belongs to $C^3(\mathcal{T})$. This is proved in Cerpa and Coron (2013, Section VI) by using the method of successive approximations. Unlikely the case of heat or wave equations, we do not have an explicit solution.

In Cerpa and Coron (2013) it is proved that the transformation (9) linking (1) and (11) is invertible, continuous and with a continuous inverse function. Therefore, the exponential decay for w, solution of (11), implies the exponential decay for the solution u controlled by (10). Thus, with this method, the following theorem is proven.

Theorem 2 (See Cerpa and Coron (2013)). *For any $\lambda>0$, there exists $C>0$ such that*

$$\|u(t,\cdot)\|_{L^2(0,L)}\le Ce^{-\lambda t}\|u(0,\cdot)\|_{L^2(0,L)},\quad \forall t\ge 0, \tag{13}$$

for any solution of (7)–(10).

We give later more details on the observer design. Let us remark that the output feedback law is designed as

$$\kappa(t):=\int_0^L k(0,y)\hat{u}(t,y)dy, \tag{14}$$

where $\hat{u}$ is the solution of (8).

Thus we get the following result, which can be compared to Marx and Cerpa (2014). The proof is given in Section 5.

Theorem 3. *For any $\lambda > 0$, there exists $C > 0$ such that for any $u_0, \hat{u}_0 \in L^2(0, L)$, the solution of (7)–(8)–(14) satisfies $\forall t \geq 0$*

$$\|u(t, \cdot) - \hat{u}(t, \cdot)\|_{L^2(0,L)} + \|\hat{u}(t, \cdot)\|_{L^2(0,L)} \leq Ce^{-\lambda t}\Big(\|u(0, \cdot) - \hat{u}(0, \cdot)\|_{L^2(0,L)} + \|\hat{u}(0, \cdot)\|_{L^2(0,L)}\Big) \tag{15}$$

Remark 2. Notice that from this theorem, we get the exponential decreasing to 0 of the L^2-norm of the solution $u = u(t, x)$. This result is different from Marx and Cerpa (2014), where the initial condition has to be chosen in $H^3(0, L)$. This is due to the fact that the output and the boundary conditions are different.

Remark 3. We see in Section 5 that the well-posedness of this linear closed-loop system directly follows from the method that we apply.

4. Observer design

The observer (8) is based on a Volterra transformation mapping the solution $\tilde{u} := u - \hat{u}$, which fulfills the following PDE

$$\begin{cases} \tilde{u}_t + \tilde{u}_x + \tilde{u}_{xxx} - p_1(x)[\tilde{u}(t, L)] = 0, \\ \tilde{u}(t, 0) = \tilde{u}_x(t, L) = \tilde{u}_{xx}(t, L) = 0, \\ \tilde{u}(0, x) = u_0(x) - \hat{u}_0(x) := \tilde{u}_0(x), \end{cases} \tag{16}$$

into the solution $\tilde{w}$ of the following PDE

$$\begin{cases} \tilde{w}_t + \tilde{w}_x + \tilde{w}_{xxx} + \lambda\tilde{w} = 0, \\ \tilde{w}(t, 0) = \tilde{w}_x(t, L) = \tilde{w}_{xx}(t, L) = 0, \\ \tilde{w}(0, x) = \tilde{w}_0(x). \end{cases} \tag{17}$$

We choose the same λ that was used to design the controller. The transformation is given by

$$\tilde{u}(x) := \Pi_o(\tilde{w}(x)) = \tilde{w}(x) - \int_x^L p(x, y)\tilde{w}(y)dy, \tag{18}$$

where p is a kernel that satisfies a partial differential equation and will be defined in the following.

In Tang and Krstic (2015), a similar observer has been designed. Indeed, if we make the change of coordinates

$$\bar{x} = L - x, \tag{19}$$

then we obtain a Korteweg–de Vries equation similar to the one studied in Tang and Krstic (2015). Hence, we know from that paper that the kernel p solves the following equation

$$\begin{cases} p_{xxx} + p_{yyy} + p_x + p_y = \lambda p, \quad \forall (x, y) \in \mathcal{T}, \\ p(x, x) = 0, \quad \forall x \in [0, L], \\ p_x(x, x) = \dfrac{\lambda}{3}(x - L), \quad \forall x \in [0, L], \\ p(0, y) = 0, \quad \forall y \in [0, L]. \end{cases} \tag{20}$$

The solution p to this equation exists and is unique. It belongs to the set $C^3(\mathcal{T})$. Moreover, once again following Tang and Krstic (2015), we obtain that

$$p_1(x) = p_{yy}(x, L) + p(x, L). \tag{21}$$

As in Tang and Krstic (2015), we can state that the transformation Π_o is invertible with continuous inverse given by

$$\tilde{w}(x) = \Pi_o^{-1}(\tilde{u}(x)) = \tilde{u}(x) + \int_x^L m(x, y)\tilde{u}(y)dy \tag{22}$$

where $m = m(x, y)$ is also a C^3 solution of an equation like (20) in the triangular domain $\mathcal{T}$.

5. Well-posedness and exponential stability of the linear system

5.1. Preliminaries

Let us focus on the linearized version of the Korteweg–de Vries equation. The homogeneous equation is given by

$$\begin{cases} u_t + u_x + u_{xxx} = 0, \\ u(t, 0) = u_x(t, L) = u_{xx}(t, L) = 0, \\ u(0, x) = u_0(x). \end{cases} \tag{23}$$

The operator associated to this linear PDE is given by:

$$\begin{aligned} &A : D(A) \subset L^2(0, L) \to L^2(0, L), \\ &w \mapsto -w' - w''' \end{aligned} \tag{24}$$

whose domain is $D(A) := \{w \in H^3(0, L) \,/\, w(0) = w'(L) = w''(L) = 0\}$. From basic semigroup theory, it is easy to prove that A generates a strongly continuous semigroup of contractions. This semigroup will be denoted by $(W(t))_{t\geq 0}$. We also need some results on the non-homogeneous Korteweg–de Vries equation.

Theorem 4 (*See Kramer, Rivas, and Zhang (2013)*)**.** *Let $T > 0$. For any $u_0 \in L^2(0, L)$ and any $(f, h) \in L^1(0, T; L^2(0, L)) \times H^{\frac{1}{3}}(0, T)$, the following Korteweg–de Vries equation*

$$\begin{cases} u_t + u_x + u_{xxx} = f, \\ u(t, 0) = h(t), \\ u_x(t, L) = u_{xx}(t, L) = 0, \\ u(0, x) = u_0(x), \end{cases} \tag{25}$$

admits a unique solution $u \in \mathcal{B}(T)$. Moreover, there exists a positive value C such that the following inequality holds:

$$\|u\|_{\mathcal{B}(T)} \leq C\Big(\|u_0\|_{L^2(0,L)} + \|f\|_{L^1(0,T;L^2(0,L))} + \|h\|_{H^{\frac{1}{3}}(0,T)}\Big) \tag{26}$$

From this result, the following lemma can be deduced.

Lemma 1. *Let us suppose all the assumptions in Theorem 4 and that in addition $f \in L^2(0, T; L^1(0, L))$. Then, one has the regularity*

$$u \in H^{\frac{1}{3}}(0, T; L^2(0, L)). \tag{27}$$

Proof. Note that in particular, $u \in L^2(0, T; H^1(0, L))$. Therefore, since $u_t = -u_x - u_{xxx} + f$, one has

$$u_t \in L^2(0, T; H^{-2}(0, L)).$$

Hence,

$$u \in H^1(0, T; H^{-2}(0, L)).$$

Then, by applying the classical theory of interpolation, it is easy to see that $u \in H^{\frac{1}{3}}(0, T; L^2(0, L))$, which concludes the proof of Lemma 1. □

Other result we need is the regularity of the right hand side in the observer.

Lemma 2. *Let $p_1 \in L^2(0, L)$. Then, given a positive value T, for every $u, \hat{u} \in L^2(0, T; H^1(0, L))$, one has $p_1(x)[u(t, L) - \hat{u}(t, L)] \in L^2(0, T; L^2(0, L))$.*

Proof. Using trace theorem, we obtain that there exists a positive value C_A such that

$$\int_0^T |u(t, L) - \hat{u}(t, L)|^2 dt \leq C_A \int_0^T \|u(t, .) - \hat{u}(t, .)\|^2_{H^1(0,L)} dt.$$

Since $u, \hat{u} \in L^2(0,T;H^1(0,L))$ and $p_1(x) \in L^2(0,L)$, we conclude the proof. □

5.2. Proof of Theorem 3 - well-posedness

The closed-loop system (7)–(8)–(14) through the transformations (9) and (18) can be written as follows

$$\begin{cases} \hat{w}_t + \hat{w}_x + \hat{w}_{xxx} + \lambda \hat{w} = \\ -\left\{ p_1(x) - \int_x^L k(x,y)p_1(y)dy \right\} \tilde{w}(t,L), \\ \hat{w}(t,0) = \hat{w}_x(t,L) = \hat{w}_{xx}(t,L) = 0, \\ \tilde{w}_t + \tilde{w}_x + \tilde{w}_{xxx} + \lambda \tilde{w} = 0, \\ \tilde{w}(t,0) = \tilde{w}_x(t,L) = \tilde{w}_{xx}(t,L) = 0. \end{cases} \tag{28}$$

We only have to prove that system (28) is well-posed (with solutions in $\mathcal{B}(T) \times \mathcal{B}(T)$ for any $T > 0$) and exponentially stable to the origin. In fact, once that is done, by using Lemma 1 we conclude that $\tilde{w}, \hat{w}$ belong to $H^{\frac{1}{3}}(0,T;L^2(0,L))$. Thus the control defined by (14) belongs to $H^{\frac{1}{3}}(0,T)$, which is the desired regularity for the input. The existence of solutions and the exponential decay for system (7)–(8)–(14) is obtained by the invertibility of the transformations (9) and (18).

System (28) is in cascade form. We can apply the linear results to get $\tilde{w}$ and then plug it into the equation for $\hat{w}$ as a right hand side by using Theorem 4. Thus, we finally get $\hat{w}$.

With these results in hand we can define the continuous solution map

$$(u_0, \hat{u}_0) \in L^2(0,L)^2 \mapsto \Lambda(u_0,\hat{u}_0) = (u,\hat{u}) \in \mathcal{B}(T)^2 \tag{29}$$

where $\mathcal{B}(T)^2 := \mathcal{B}(T) \times \mathcal{B}(T)$ and $u, \hat{u} \in \mathcal{B}(T)$ are the solutions of the linear closed loop system (7)–(8)–(14).

5.3. Proof of Theorem 3 - stability

Let us focus now on the exponential stability. To do that, we consider the following Lyapunov function

$$V(t) := V_1(t) + V_2(t) \tag{30}$$

where

$$V_1(t) = A\int_0^L \hat{w}(t,x)^2 dx, \tag{31}$$

and

$$V_2(t) = B\int_0^L \tilde{w}(t,x)^2 dx. \tag{32}$$

The positive values A and B are chosen later. After performing some integrations by parts, we obtain

$$\dot{V}_1(t) \leq \left(-2\lambda + \frac{D^2}{A}\right) V_1(t) + A^2 L|\tilde{w}(t,L)|^2, \tag{33}$$

where

$$D = \max_{x\in[0,L]} \left\{ p_1(x) - \int_x^L k(x,y)p_1(y)dy \right\}. \tag{34}$$

We have also

$$\dot{V}_2(t) \leq -2\lambda V_2(t) - B|\tilde{w}(t,L)|^2. \tag{35}$$

Therefore, by choosing

$$A > \frac{D^2}{2\lambda} \tag{36}$$

and

$$B \geq A^2 L, \tag{37}$$

we get

$$\dot{V}(t) \leq -2\mu V(t), \tag{38}$$

where

$$\mu = \left(\lambda - \frac{D^2}{2A}\right) > 0. \tag{39}$$

Thus, we obtain the exponential decay of $(\tilde{w}, \hat{w})$ with a decay rate equals to μ. By using the invertibility and continuity of the transformations Π and Π_o, we conclude the proof of Theorem 3 by getting the desired exponential decay for $(\tilde{u}, \hat{u})$.

6. Well-posedness and exponential stability of the nonlinear system

6.1. Preliminaries

The following results are useful.

Lemma 3 (*See Rosier (1997)*). *Given a positive value T, let $u \in L^2(0,T;H^1(0,L))$. Then $uu_x \in L^1(0,T;L^2(0,L))$ and the map $u \in L^2(0,T;H^1(0,L)) \mapsto uu_x \in L^1(0,T;L^2(0,L))$ is continuous. Moreover, there exists a positive value C_h such that, for every $u, \tilde{u} \in L^2(0,T;H^1(0,L))$*

$$\begin{aligned} &\|uu_x - \tilde{u}\tilde{u}_x\|_{L^1(0,T;L^2(0,L))} \\ &\quad \leq C_h \|u + \tilde{u}\|_{L^2(0,T;H^1(0,L))} \|u - \tilde{u}\|_{L^2(0,T;H^1(0,L))} \end{aligned} \tag{40}$$

Lemma 4 (*See Kramer et al. (2013)*). *For any positive value T, there exist two positive values $C_1 := C_1(T)$ and $C_2 := C_2(T)$ such that*

(i) *For any $z, \hat{z} \in \mathcal{B}(T)$,*

$$\int_0^T \|(z(t,.)\hat{z}(t,.))_x\|_{L^2(0,L)} dt \leq C_1 \|z\|_{\mathcal{B}(T)} \|\hat{z}\|_{\mathcal{B}(T)} \tag{41}$$

(ii) *For $f \in L^1(0,T;L^2(0,L))$, let*

$$u = \int_0^t W(t-s)f(s)ds,$$

then

$$\|u\|_{\mathcal{B}(T)} \leq C_2 \int_0^T \|f(t,.)\|_{L^2(0,L)} dt. \tag{42}$$

Moreover, one can easily prove the following lemma.

Lemma 5. *Given a positive value T, let $u \in \mathcal{B}(T)$. Then $uu_x \in L^2(0,T;L^1(0,L))$. Moreover, for every $u \in \mathcal{B}(T)$*

$$\begin{aligned} &\|uu_x\|_{L^2(0,T;L^1(0,L))} \\ &\quad \leq \|u\|_{L^2(0,T;H^1(0,L))} \|u\|_{C([0,T];L^2(0,L))} \end{aligned} \tag{43}$$

6.2. Proof of Theorem 1 - well-posedness

We will apply the Banach fixed point theorem in order to prove the well-posedness of the nonlinear closed-loop system.

With κ defined by (14), and $z, \hat{z} \in \mathcal{B}(T)$, the solutions to

$$\begin{cases} u_t + u_x + u_{xxx} = -zz_x, \\ u(t,0) = \kappa(t), \\ u_x(t,L) = u_{xx}(t,L) = 0, \\ u(0,x) = u_0(x), \end{cases} \tag{44}$$

and

$$\begin{cases} \hat{u}_t + \hat{u}_x + \hat{u}_{xxx} + p_1(x)[u(t,L) - \hat{u}(t,L)] = -\hat{z}\hat{z}_x, \\ \hat{u}(t,0) = \kappa(t), \\ \hat{u}_x(t,L) = \hat{u}_{xx}(t,L) = 0, \\ \hat{u}(0,x) = \hat{u}_0(x), \end{cases} \tag{45}$$

can be written as follows

$$(u,\hat{u})(t)=\Lambda(u_0,\hat{u}_0)(t)-\Big(\int_0^t W(t-s)z(s)z_x(s)ds,\int_0^t W(t-s)\hat{z}(s)\hat{z}_x(s)ds\Big) \tag{46}$$

where Λ was introduced in (29) and W is the semigroup associated to the linear single equation. We will be done if we prove that the map Γ defined by the right hand-side of (46),

$$\Gamma(z,\hat{z})=\Lambda(u_0,\hat{u}_0)-\Big(\int_0^t W(t-s)z(s)z_x(s)ds,\int_0^t W(t-s)\hat{z}(s)\hat{z}_x(s)ds\Big), \tag{47}$$

has a fixed point.

We define, for some $R>0$ to be chosen later,

$$S_R=\Big\{(z,\hat{z})\in\mathcal{B}(T)\times\mathcal{B}(T)\,/\,\|z\|_{\mathcal{B}(T)}+\|\hat{z}\|_{\mathcal{B}(T)}\le R\Big\}, \tag{48}$$

which is a closed convex and bounded subset of $\mathcal{B}(T)^2$. Consequently, S_R is a complete metric space in the topology induced by $\mathcal{B}(T)^2$. With Theorem 4 and previous lemmas, we have the existence of a constant $C>0$ such that for any $(z,\hat{z})\in S_R$,

$$\|\Gamma(z,\hat{z})\|_{\mathcal{B}(T)^2}\le C\Big(\|u_0\|_{L^2(0,L)}+\|\hat{u}_0\|_{L^2(0,L)}\Big)+C\Big(\|z\|^2_{\mathcal{B}(T)}+\|\hat{z}\|^2_{\mathcal{B}(T)}\Big). \tag{49}$$

We consider $r>0$ and $u_0,\hat{u}_0\in L^2(0,L)$ such that

$$\|u_0\|_{L^2(0,L)}\le r,\quad \text{and}\quad \|\hat{u}_0\|_{L^2(0,L)}\le r.$$

Now we select r,R as follows

$$\begin{cases} 2Cr\le R/2,\\ 2CR^2\le R/2,\end{cases} \tag{50}$$

to obtain that for any $(z,\hat{z})\in S_R$

$$\|\Gamma(z,\hat{z})\|_{\mathcal{B}(T)^2}\le R. \tag{51}$$

Thus, with such r and R, Γ maps S_R into S_R. Moreover, using Lemmas 3 and 4, we obtain

$$\begin{aligned}\|\Gamma(z_1,\hat{z}_1)-\Gamma(z_2,\hat{z}_2)\|_{\mathcal{B}^2(T)}&\le C_2C_h\Big(\|z_1+z_2\|_{\mathcal{B}(T)}\|z_1-z_2\|_{\mathcal{B}(T)}\\&\quad+\|\hat{z}_1+\hat{z}_2\|_{\mathcal{B}(T)}\|\hat{z}_1-\hat{z}_2\|_{\mathcal{B}(T)}\Big)\end{aligned} \tag{52}$$

and thus, choosing R such that

$$2RC_2C_h\le\frac{1}{2}$$

we arrive at

$$\|\Gamma(z_1,\hat{z}_1)-\Gamma(z_2,\hat{z}_2)\|_{\mathcal{B}^2(T)}\le\frac{1}{2}\Big(\|z_1-z_2\|_{\mathcal{B}(T)}+\|\hat{z}_1-\hat{z}_2\|_{\mathcal{B}(T)}\Big) \tag{53}$$

for any $(z_1,\hat{z}_1),(z_2,\hat{z}_2)\in S_R$. By using the Banach fixed-point theorem (see Brezis (2011, Theorem 5.7)) we obtain the existence of a unique fixed point of the map Γ. As previously stated, this fixed point is the solution we are looking for.

6.3. Proof of Theorem 1 - stability

As we already have solutions, the aim now is to prove the local exponential stability of the nonlinear closed-loop system

$$\begin{cases} u_t+u_x+u_{xxx}+u_xu=0,\\ u(t,0)=\kappa(t),\ u_x(t,L)=u_{xx}(t,L)=0,\\ \hat{u}_t+\hat{u}_x+\hat{u}_{xxx}+\hat{u}_x\hat{u}+p_1(x)[y(t)-\hat{u}(t,L)]=0,\\ \hat{u}(t,0)=\kappa(t),\ \hat{u}_x(t,L)=\hat{u}_{xx}(t,L)=0,\end{cases} \tag{54}$$

where

$$\kappa(t)=\int_0^L k(0,y)\hat{u}(t,y)dy, \tag{55}$$

and

$$y(t)=u(t,L). \tag{56}$$

As before, we consider the evolution of the couple $(\tilde{u},\hat{u})$ where $\tilde{u}$ stands for the error $\tilde{u}=u-\hat{u}$. Using Π_o and its inverse (see (18) and (22)), we define $\tilde{w}=\Pi_o^{-1}(\tilde{u})$. We denote $\hat{w}=\Pi(\hat{u})$, where Π is defined in (9). The inverse Π^{-1} is given by

$$\hat{u}(x)=\Pi^{-1}(\hat{w}(x))=\hat{w}(x)+\int_x^L l(x,y)\hat{w}(y)dy, \tag{57}$$

where l solves the following equation

$$\begin{cases} l_{xxx}+l_{yyy}+l_x+l_y=\lambda l, & \text{in } \mathcal{T},\\ l(x,L)+l_{yy}(x,L)=0, & \text{in } [0,L],\\ l(x,x)=0, & \text{in } [0,L],\\ l_x(x,x)=\dfrac{\lambda}{3}(L-x), & \text{in } [0,L].\end{cases} \tag{58}$$

The existence of such a kernel l has been proven in Cerpa and Coron (2013, see sections IV and VI). Thus, we can see that $(\tilde{u},\hat{u})$ is mapped into $(\tilde{w},\hat{w})$ solution of the coupled target system

$$\begin{aligned}\hat{w}_t(t,x)&+\hat{w}_x(t,x)+\hat{w}_{xxx}(t,x)+\lambda\hat{w}(t,x)\\&=-\Big\{p_1(x)-\int_x^L k(x,y)p_1(y)dy\Big\}\tilde{w}(t,L)\\&\quad-\Big(\hat{w}(t,x)+\int_x^L l(x,y)\hat{w}(t,y)dy\Big)\\&\quad\cdot\Big(\hat{w}_x(t,x)+\int_x^L l_x(x,y)\hat{w}(t,y)dy\Big)\\&\quad-\frac{1}{2}\int_x^L|\hat{u}(t,y)|^2k_y(x,y)dy,\end{aligned} \tag{59}$$

$$\begin{aligned}\tilde{w}_t(t,x)&+\tilde{w}_x(t,x)+\tilde{w}_{xxx}(t,x)+\lambda\tilde{w}(t,x)\\&=-\Big(\tilde{w}(t,x)-\int_x^L p(x,y)\tilde{w}(t,y)dy\Big)\\&\quad\cdot\Big(\tilde{w}_x(t,x)-\int_x^L p_x(x,y)\tilde{w}(t,y)dy\Big)\\&\quad-\Big(\hat{w}(t,x)+\int_x^L l(x,y)\hat{w}(t,y)dy\Big)\\&\quad\cdot\Big(\tilde{w}_x(t,x)-\int_x^L p_x(x,y)\tilde{w}(t,y)dy\Big)\\&\quad-\Big(\tilde{w}(t,x)-\int_x^L p(x,y)\tilde{w}(t,y)dy\Big)\\&\quad\cdot\Big(\hat{w}_x(t,x)+\int_x^L l_x(x,y)\hat{w}(t,y)dy\Big)\\&\quad+\int_x^L\Big[\frac{|\tilde{u}(t,y)|^2}{2}+\tilde{u}(t,y)\hat{u}(t,y)\Big]m_y(x,y)dy,\end{aligned} \tag{60}$$

with boundary conditions

$$\hat{w}(t,0)=\hat{w}_x(t,L)=\hat{w}_{xx}(t,L)=0, \tag{61}$$

$$\tilde{w}(t,0) = \tilde{w}_x(t,L) = \tilde{w}_{xx}(t,L) = 0. \tag{62}$$

As in previous section, we will prove the stability of this system by using the same Lyapunov function (30). We derivate (31) with respect to time as follows

$$\begin{aligned}\dot{V}_1(t) = {} & 2A\int_0^L \hat{w}_t(t,x)\hat{w}(t,x)dx \\ \leq {} & \left(-2\lambda + \frac{D^2}{A}\right)V_1(t) + A^2L|\tilde{w}(t,L)|^2 \\ & - 2A\int_0^L \hat{w}(t,x)F(t,x)dx,\end{aligned} \tag{63}$$

where

$$\begin{aligned}F(t,x) = {} & \hat{w}(t,x)\hat{w}_x(t,x) + \hat{w}(t,x)\int_x^L l_x(x,y)\hat{w}(t,y)dy \\ & + \hat{w}_x(t,x)\int_x^L l(x,y)\hat{w}(t,y)dy \\ & + \left(\int_x^L l(x,y)\hat{w}(t,y)dy\right)\left(\int_x^L l_x(x,y)\hat{w}(t,y)dy\right) \\ & + \frac{1}{2}\int_x^L |\hat{u}(t,y)|^2 k_y(x,y)dy.\end{aligned} \tag{64}$$

By using the same argument as in Cerpa and Coron (2013) and Coron and Lü (2014), we can prove the existence of a positive constant $K_1 = K_1(\|l\|_{C^1(\mathcal{T})}, \|k\|_{C^1(\mathcal{T})})$ such that

$$\left|A\int_0^L \hat{w}(t,x)F(t,x)dx\right| \leq K_1\left(\int_0^L |\hat{w}(t,x)|^2dx\right)^{\frac{3}{2}}. \tag{65}$$

Then, we estimate $\dot{V}_2(t)$ as follows

$$\begin{aligned}&\dot{V}_2(t) \leq -2\lambda V_2(t) \\ &\quad - 2B\int_0^L \tilde{w}(t,x)G(t,x)dx - B|\tilde{w}(t,L)|^2\end{aligned} \tag{66}$$

where $G = G(t,x)$ is the right-hand side of (60). As before, we can prove the existence of a positive constant $K_2 = K_2(\|l\|_{C^1(\mathcal{T})}, \|p\|_{C^1(\mathcal{T})}, \|m\|_{C^1(\mathcal{T})})$ such that

$$\begin{aligned}&\left|2B\int_0^L \tilde{w}(t,x)G(t,x)dx\right| \leq K_2\left(\int_0^L |\hat{w}(t,x)|^2dx\right)^{\frac{3}{2}} \\ &\quad + K_2\left(\int_0^L |\tilde{w}(t,x)|^2dx\right)^{\frac{3}{2}}.\end{aligned} \tag{67}$$

Therefore, for A, B, μ satisfying (36)–(37)–(39), we have

$$\begin{aligned}\dot{V}(t) \leq {} & -2\mu V(t) + K_1\left(\int_0^L |\hat{w}(t,x)|^2dx\right)^{\frac{3}{2}} \\ & + K_2\left(\int_0^L |\hat{w}(t,x)|^2dx\right)^{\frac{3}{2}} \\ & + K_2\left(\int_0^L |\tilde{w}(t,x)|^2dx\right)^{\frac{3}{2}}.\end{aligned} \tag{68}$$

If there exists $t_0 \geq 0$ such that

$$\|\tilde{w}(t_0,.)\|_{L^2(0,L)} \leq \frac{\mu}{K_2} \tag{69}$$

and

$$\|\hat{w}(t_0,.)\|_{L^2(0,L)} \leq \frac{\mu}{K_1+K_2} \tag{70}$$

we can conclude

$$\dot{V}(t) \leq -\mu V(t), \quad \forall t \geq t_0. \tag{71}$$

Thus, we get

$$\begin{aligned}&\|\tilde{w}(t,.)\|_{L^2(0,L)} + \|\hat{w}(t,.)\|_{L^2(0,L)} \leq \\ &\quad e^{-\frac{\mu}{2}t}\left(\|\tilde{w}_0\|_{L^2(0,L)} + \|\hat{w}_0\|_{L^2(0,L)}\right), \quad \forall t \geq 0,\end{aligned} \tag{72}$$

provided that

$$\begin{aligned}&\|\hat{w}_0\|_{L^2(0,L)} \leq \frac{\mu}{K_1+K_2}, \\ &\|\tilde{w}_0\|_{L^2(0,L)} \leq \frac{\mu}{K_2}.\end{aligned} \tag{73}$$

This is the local exponential stability for $(\tilde{w}, \hat{w})$. As in the linear case, we conclude the proof of Theorem 1 by going back to the solutions $(\tilde{u}, \hat{u})$. This is done with the continuous and invertible transformations Π and Π_o. The exponential decay of the system is obtained provided a smallness condition on the L^2-norm of the initial data $u_0, \hat{u}_0$ holds.

7. Numerical simulations

In this section we provide some numerical simulations showing the effectiveness of our control design. In order to discretize our KdV equation, we use a finite difference scheme inspired from Pazoto, Sepúlveda, and Vera Villagrán (2010). The final time for simulations is denoted by T_{final}. We choose $(N_x + 1)$ points to build a uniform spatial discretization of the interval $[0, L]$ and $(N_t + 1)$ points to build a uniform time discretization of the interval $[0, T_{\text{final}}]$. Thus, the space step is $\Delta x = L/N_x$ and the time step $\Delta t = T_{\text{final}}/N_t$. We approximate the solution with the notation $u(t,x) \approx U_j^i$, where i and j refer to time and space discrete variables, respectively.

Some used approximations of the derivative are given by

$$u_x(t,x) \approx \nabla_-(U_j^i) = \frac{U_j^i - U_{j-1}^i}{\Delta x} \tag{74}$$

or

$$u_x(t,x) \approx \nabla_+(U_j^i) = \frac{U_{j+1}^i - U_j^i}{\Delta x}. \tag{75}$$

As in Pazoto et al. (2010), we choose rather the following

$$u_x(t,x) \approx \frac{1}{2}(\nabla_+ + \nabla_-)(U_j^i) = \nabla(U_j^i). \tag{76}$$

For the other differentiation operator, we use

$$u_{xxx}(t,x) \approx \nabla_+\nabla_+\nabla_-(U_j^i) \tag{77}$$

and

$$u_t(t,x) \approx \frac{U_j^{i+1} - U_j^i}{\Delta t}. \tag{78}$$

Let us introduce a matrix notation. Let us consider $D_-, D_+, D \in \mathbb{R}^{N_x \times N_x}$ given by

$$D_- = \frac{1}{\Delta x}\begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ -1 & 1 & \ddots & & \vdots \\ 0 & -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix}, \tag{79}$$

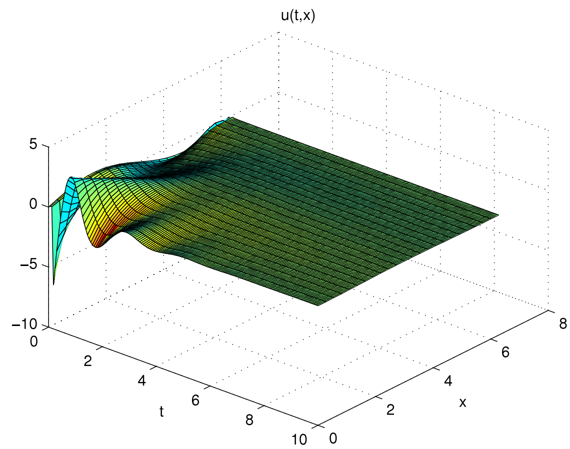

Fig. 1. Solution of the closed-loop system (1)–(3)–(4)–(14).

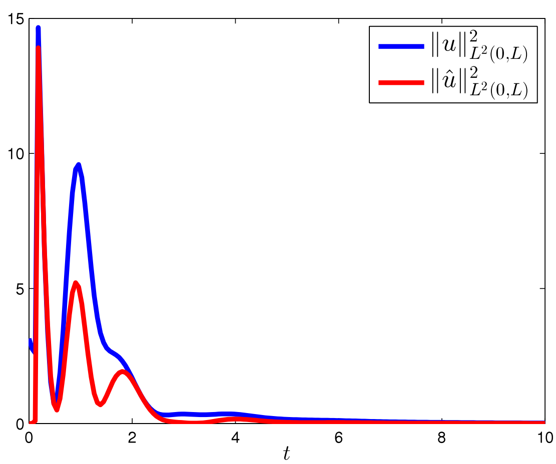

Fig. 2. Time evolution of the L^2-norm for the state (blue line) and the observer (red line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$D_+ = \frac{1}{\Delta x}\begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & 0 & -1 & 1 \\ 0 & \dots & \dots & 0 & -1 \end{bmatrix}, \qquad (80)$$

and $D = (D_+ + D_-)/2$. Let us define $A := D_+D_+D_- + D$, and $C := A + \Delta t I_d$ where I_d is the identity matrix. Moreover, we will denote, for each discrete time i,

$$U^i := \begin{bmatrix} U_1^i & U_2^i & \dots & U_{N_x+1}^i \end{bmatrix}^T$$

the plant state, and

$$O^i := \begin{bmatrix} O_1^i & O_2^i & \dots & O_{N_x+1}^i \end{bmatrix}^T$$

the observer state. The state output will be denoted by Y_U and Y_O stands for the observer output. The discretized controller gain K and observer gain P, respectively, are defined by

$$K = \begin{bmatrix} K_1 & K_2 & \dots & K_{N_x+1} \end{bmatrix}^T$$

and

$$P = \begin{bmatrix} P_1 & P_2 & \dots & P_{N_x+1} \end{bmatrix}^T.$$

We compute them from a successive approximations method (see Cerpa and Coron (2013)). Since we have the nonlinearity uu_x, we use an iterative fixed point method to solve the nonlinear system

$$CU^{i+1} = U^i - \frac{1}{2}D(U^{i+1})^2.$$

With $N_{iter} = 5$, which denotes the number of iterations of the fixed point method, we get good approximations of the solutions.

Given $U^0, O^0 = 0, K$, and P, the following is the structure of the algorithm used in our simulations.

While $i < N_t$

- $U_1^{i+1} = O_1^{i+1} = \sum_{j=1}^{N_x} \Delta x \frac{O_{j+1}^i K_{j+1} + O_j^i K_j}{2}$
  $U_{N_x}^{i+1} = U_{N_x+1}^{i+1} = U_{N_x-1}^{i+1},\ O_{N_x}^{i+1} = O_{N_x+1}^{i+1} = O_{N_x+1}^{i+1};$
- $Y_U := U^i(N_x + 1), Y_O := O^i(N_x + 1);$
- Setting $J(1) = O^i$, for all $k \in \{1, \dots, N_{iter}\}$, solve
  $$J(k+1) = C^{-1}(O^i - \frac{1}{2}D(J(k))^2 + P(Y_U - Y_O))$$
  Set $O^{i+1} = J(N_{iter});$
- Setting $\tilde{J}(1) = U^i$, for all $k \in \{1, \dots, N_{iter}\}$, solve
  $$\tilde{J}(k+1) = C^{-1}(U^i - \frac{1}{2}D(\tilde{J}(k))^2)$$
  Set $U^{i+1} = J(N_{iter});$
- $t = t + dt;$

End

In order to illustrate our theoretical results, we perform some simulations on the domain $[0, 2\pi]$. We take $N_x = 30$, $N_t = 167$, $T_{\text{final}} = 10$, $\lambda = 2$, $u_0(x) = \sin(x)$ and $\hat{u}_0(x) = 0$. Fig. 1 illustrates the convergence to the origin of the solution of the closed-loop system (1)–(3)–(4)–(14). Fig. 2 illustrates the L^2-norm of this solution and the L^2-norm of the solution of the observer (4). Finally, Fig. 3 illustrates the logarithmic time evolution of the L^2-norm of the observation error $(u - \hat{u})$ and of $(u_0 - \hat{u}_0)e^{-\mu t}$ where $\mu = 0.4$. Note that the observation error converges to 0 in L^2-norm. From the simulations, this convergence seems to be exponential as expected.

8. Conclusions

In this paper, an output feedback control has been designed for the Korteweg–de Vries equation. This controller uses an observer and gives the local exponential stability of the closed-loop system. Numerical simulations have been provided to illustrate the efficiency of the output feedback design.

In order to go to a global feedback control for the nonlinear KdV equation, a first step should be to build some nonlinear boundary controls giving a semi-global exponential stability. That means that for any fixed $r > 0$ we can find a feedback law exponentially

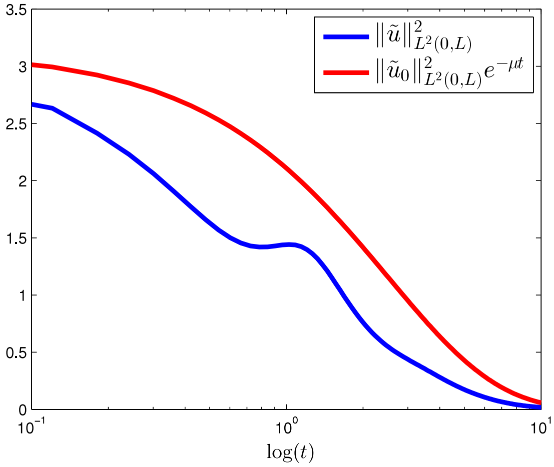

Fig. 3. Logarithmic time evolution of the L^2-norm for the observation error $\tilde{u} = (u - \hat{u})$ and $\|\tilde{u}_0\|^2_{L^2(0,L)}e^{-\mu t}$.

stabilizing to the origin any solution with initial data u_0 whose L^2-norm is smaller than r. The term semi-global comes from the fact that the decay rate can depend on r. Some internal feedback controls are given for KdV in Marx et al. (2017), Pazoto (2005) and Rosier and Zhang (2006). The latter considers saturated controls.

The second step, in order to deal with the output case, is to design an observer for the nonlinear KdV equation to obtain a global asymptotic stability. We could for instance follow the same approach performed in the finite-dimensional case in Gauthier, Hammouri, and Othman (1992).

References

Brezis, Haim (2011). Functional analysis, Sobolev spaces and partial differential equations. In *Universitext* (p. xiv+599). New York: Springer.

Carreño, N., & Guerrero, S. (2015). On the non-uniform null controllability of a linear KdV equation. *Asymptotic Analysis*, *94*(1–2), 33–69.

Cerpa, E. (2014). Control of a Korteweg-de Vries equation: a tutorial. *Mathematical Control and Related Fields*, *4*(1), 45–99.

Cerpa, E., & Coron, J.-M. (2013). Rapid stabilization for a Korteweg-de Vries equation from the left Dirichlet boundary condition. *IEEE Transactions on Automatic Control*, *58*(7), 1688–1695.

Cerpa, E., & Crépeau, E. (2009). Rapid exponential stabilization for a linear Korteweg-de Vries equation. *Discrete and Continuous Dynamical Systems. Series B*, *11*(3), 655–668.

Cerpa, E., Rivas, I., & Zhang, B.-Y. (2013). Boundary controllability of the Korteweg-de Vries equation on a bounded domain. *SIAM Journal on Control and Optimization*, *51*(4), 2976–3010.

Coron, J.-M., & Lü, Q. (2014). Local rapid stabilization for a Korteweg-de Vries equation with a Neumann boundary control on the right. *Journal de Mathématiques Pures et Appliquées. Neuviéme Série*, *102*(6), 1080–1120.

Gauthier, J.-P., Hammouri, H., & Othman, S. (1992). A simple observer for nonlinear systems applications to bioreactors. *IEEE Transactions on Automatic Control*, *37*(6), 875–880.

Glass, O., & Guerrero, S. (2008). Some exact controllability results for the linear KdV equation and uniform controllability in the zero-dispersion limit. *Asymptotic Analysis*, *60*(1–2), 61–100.

Guilleron, J.-P. (2014). Null controllability of a linear kdV equation on an interval with special boundary conditions. *Mathematics of Control, Signals, and Systems*, *26*(3), 375–401.

Hasan, A. (2016). Output-feedback stabilization of the Korteweg de-Vries equation. In *Mediterranean conference on control and automation* (pp. 871–876). Athens, Greece.

Kramer, Eugene, Rivas, Ivonne, & Zhang, Bing-Yu (2013). Well-posedness of a class of non-homogeneous boundary value problems of the Korteweg-de Vries equation on a finite domain. *ESAIM. Control, Optimisation and Calculus of Variations*, *19*(2), 358–384.

Krstic, M., & Smyshlyaev, A. (2008). Boundary control of PDEs. A course on backstepping designs. In *Advances in design and control*: *Vol. 16*. (p. x+192). Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM).

Marx, S., & Cerpa, E. (2014). Output feedback control of the linear Korteweg-de Vries equation. In *53rd IEEE conference on decision and control* (pp. 2083–2087). Los Angeles, USA.

Marx, Swann, Cerpa, Eduardo, Prieur, Christophe, & Andrieu, Vincent (2017). Global stabilization of a Korteweg–de Vries equation with saturating distributed control. *SIAM Journal on Control and Optimization*, *55*(3), 1452–1480.

Pazoto, A. F. (2005). Unique continuation and decay for the korteweg-de Vries equation with localized damping. *ESAIM. Control, Optimisation and Calculus of Variations*, *11*(3), 473–486 (electronic).

Pazoto, A. F., Sepúlveda, M., & Vera Villagrán, O. (2010). Uniform stabilization of numerical schemes for the critical generalized Korteweg-de Vries equation with damping. *Numerische Mathematik*, *116*(2), 317–356.

Rosier, L. (1997). Exact boundary controllability for the Korteweg-de Vries equation on a bounded domain. *ESAIM. Control, Optimisation and Calculus of Variations.*, *2*, 33–55 (electronic).

Rosier, Lionel, & Zhang, Bing-Yu (2006). Global stabilization of the generalized Korteweg-de Vries equation posed on a finite domain. *SIAM Journal on Control and Optimization*, *45*(3), 927–956.

Rosier, L., & Zhang, B.-Y. (2009). Control and stabilization of the Korteweg-de Vries equation: recent progresses. *Journal of Systems Science & Complexity*, *22*(4), 647–682.

Smyshlyaev, A., & Krstic, M. (2005). Backstepping observers for a class of parabolic PDEs. *Systems & Control Letters*, *54*(7), 613–625.

Tang, S., & Krstic, M. (2013). Stabilization of linearized Korteweg-de Vries systems with anti-diffusion. In *American control conference* (pp. 3302–3307). Washington DC, USA.

Tang, S., & Krstic, M. (2015). Stabilization of linearized Korteweg-de Vries with anti-diffusion by boundary feedback with non-collocated observation. In *American control conference* (pp. 1959–1964). Chicago, USA.

Teel, A., & Praly, L. (1994). Global stabilizability and observability imply semi-global stabilizability by output feedback. *Systems & Control Letters*, *22*(5), 313–325.

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